Geometric Modeling
Assignment sheet #2
"Math recap" (due May 8th 2012 *before* the lecture)

Silke Jansen, Ruxandra Lasowski,
Avinash Sharma, Art Tevs, Michael Wand

Exercise 1 (Gram-Schmidt Orthogonalization): [2+2+1 points]

a. Calculate an orthogonal basis for $\mathbb{R}^3$ from the vectors

$$v_1 = \begin{pmatrix}1\\1\\1\end{pmatrix}, v_2 = \begin{pmatrix}0\\2\\0\end{pmatrix}, v_3 = \begin{pmatrix}1\\0\\3\end{pmatrix}$$

using Gram-Schmidt Orthogonalization.

b. Calculate an orthogonal basis for the functional basis $[\,1\,, x\,, x^2]$ on the interval $[0\,,1]$ using Gram-Schmidt Orthogonalization. (Remember to use the inner product for function spaces)

c. Sketch the resulting functions from b.

Exercise 2 (Basis Functions): [2+1+2 points]

a. Write down the system of linear equations that has to be solved to obtain the least-squares approximation of $f \in V$ on using the basis functions $b_1, b_2, \cdots, b_n \in V$.

b. How does a. simplify if $b_1, b_2, \cdots, b_n$ form an orthogonal basis?

c. Find the least-squares approximation of $\frac{1}{2}\, x^3 - \frac{1}{2} x$ on the interval $[0\,,1]$ using functional basis $\left[1\,, x - \frac{1}{2}\,, x^2 - x + \frac{1}{6}\right]$.

Exercise 3 (Eigenvectors and –values): [2+2+3+1+2 points]

a. Show that $u \in \mathbb{R}^n$ is an eigenvector of the matrix uu^t and has an eigenvalue of $\|u\|^2$.

b. Show that uu^t has only one non-zero eigenvalue.

c. Given a vector $v \in \mathbb{R}^n$ with $\|v\|^2 = 1$ show that $\mathbf{I} - vv^t$ has two eigenvalues with value 1. (Hint: What are eigenvectors of $\mathbf{I}$?)

d. Given eigenvectors $e_1, e_2, e_3 \in \mathbb{R}^3$ with corresponding eigenvalues d_1, d_2, d_3 , how do you reconstruct the source matrix $M \in \mathbb{R}^{3\times 3}$ from which they were calculated?

e. The Fibonacci sequence can be written as:

$$\begin{pmatrix}F_n\\F_{n+1}\end{pmatrix} = \begin{pmatrix}0 & 1\\1 & 1\end{pmatrix}^n \begin{pmatrix}F_0\\F_1\end{pmatrix}$$

Derive a closed form solution for the *n*th Fibonacci number F_n.

(Hint: Diagonal form TDT^{-1} of $\begin{pmatrix}0 & 1\\1 & 1\end{pmatrix}$ is $T = \begin{pmatrix}1 & 1\\ \frac{1+\sqrt{5}}{2} & \frac{1-\sqrt{5}}{2}\end{pmatrix}, D = \begin{pmatrix}\frac{1+\sqrt{5}}{2} & 0\\ 0 & \frac{1-\sqrt{5}}{2}\end{pmatrix}$).